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Numerical Modelling and Mathematics of Ground Water Recharging – Unconfined Aquifer

PRATIMA PATEL[1] and M. D. DESAI[2]

[1]Civil Engineering Department, Sarvajanik College of Engineering & Technology, Athwalines, Surat, Gujarat, India,

[2]Applied Mechanics Department, S. V. National Institute of Technology, Surat, Gujarat, India

Email: *pratima13p@gmail.com*

Abstract: Scarcity and Misuse of fresh water pose a serious and growing threat to sustainable development and protection of the environment. Human health and welfare, food security, industrial development and the ecosystems on which they depend, are all at risk, unless water and land resources are managed more effectively in the present decade and beyond. With the growing demand of water, recharging of aquifer is the only answer for the water crisis. This paper focus on mathematical modelling of ground water flow related to unconfined aquifer with a change in saturated thickness with variation in piezometric level s_o, permeability k, radius of influences or distance between two recharge well L and presence of recharge by rainfall P is discussed here. By using quadratic mathematical expression the drawdown can also be determined This technique is implemented to an unconfined aquifer with horizontal impervious base receiving vertical recharge using rain water stored in specially designed basin. Added would be the tremendous pressure to meet water requirements for other purposes such as for industrial use, environment and ecological management etc. emanating from population growth, the land use policies, degradation of water resources and depletion of aquifers in the country.

Authors have set up various case studies for recharge scheme which are adopted at site and gives satisfactory results, few are highlighted. Authors have established correlations between radius of bore well r and depth of pervious strata h with capacity of borehole Q_r.

Keywords: *unconfined aquifer, artificial recharge, numerical modelling, permeability, radius of influences, draw down.*

1. Introduction:

Conservation is an act of preserving resources from decay, loss or injury otherwise to handle the resources with care and safeguard against destruction. Water is one of the renewable resources. India with an average rainfall of 1150 mm is the second wettest country in the world with good water resources. But the water resources are not evenly distributed over the country due to varied Hydrogeological conditions and high variations in precipitation both in time and space. As large quantities of rainfall are going to sea as runoff, it is better to harness this wasteful runoff by adopting proper scientific conservation measures and constructing suitable recharge structures at appropriate locations and artificially recharge the depleted aquifers through recharge bore wells. About one-fifth of the world's population lacks access to safe drinking water and with the present consumption patterns; two out of every three persons on the earth would live in water-stressed conditions by 2025.Industrial, population growth and increasing agricultural activities often lead to over-exploitation of local ground-water resources in order to meet the rising demand for water. Lowered water-tables can lead to many different problems such as decrease in the water reserves and intrusion of contaminated water in bodies of potable water. This problem can be prevented by inducing ground-water mounding through artificial recharge using rain water stored in specially constructed basins. In order to maintain the regional water balance and to assure optimal use of available water, knowledge of the water-table fluctuation in response to the proposed recharged scheme is essential. A long term perspective planning of water resources is required to meet various competing demands on sustainable basis. In all these studies the rate of recharge is constant. In practice however, the rate of recharge is largely dependent on the infiltration rate which is known to decrease with time in a more-or-less exponential form mainly due to the clogging of the soil pores at the bottom of the basin [1]

2. Theoretical Aspect of Recharge Well for Ground Water Recharging:

Recharge of groundwater through storm run off and roof top. Water collection, diversion and collection of run off into dry tanks, play grounds, parks and other vacant places are to be implemented by a recharge well. Its flow is the reverse of the pumping well but its construction may or may not be the same. If water is passed into a recharge well, a cone of recharge will be formed which is reverse of a cone of depression for a pumping well .Now, steady state equations for recharge rate Q_r into a completely penetrating well for unconfined aquifer in Fig. 1 [5]

$$Q_r = \frac{\pi k\,(h_w^2 - h_0^2)}{\ln(r_0 / r_w)}$$

Though the equations are similar to discharge equations but the recharge rates are seldom equal to pumping rates .In this recharge well injecting the water (from roof top or storm water reuse) into bore holes, the water is therefore fed into recharge wells by gravity or may be pumped under pressure to increase recharge rate, if surface conditions permit. The recharge wells are just like ordinary production wells. The ordinary production wells are many a times directly used for recharge during the off season, when the water is not required for use. (Fig. 2).

- Discharge (recharge) quantity
- $Q = \dfrac{\pi k(h^2 - h_w^2)}{\ln(L/r)}$
- The phreatic surface
- $y^2 - h_w^2 = \dfrac{Q}{\pi k}\ln\left(\dfrac{x}{r}\right)$
- Equation for drawdown (recharge) curve
- $h^2 - y^2 = \dfrac{L-x}{L}(h^2 - h_w^2)$
- Estimating the free discharge height
- $h_s = \begin{cases} 0.5(h-h_0) & \text{(Boulton)} \\ \dfrac{(h-h_0)^2}{2h} & \text{(Ehrenberg)} \\ (h-h_0)e^{(-\alpha/\pi)} & \text{(Juhasz)} \end{cases}$
- $Where\ \alpha = \dfrac{\sqrt{k}}{15}\,\dfrac{2\pi\, r\, h_0}{Q}$

For design of pumping or recharge well 'L' should be known with reasonable accuracy as it is indirectly proportional to 'q_o' or 'q_r'. In the well formula 'L' appears in a logarithmic term and a good assessment of co-efficient of permeability (k) is much more important than the value of influence range (L).

3. Mathematical Formulation Relevant To Ground Water Recharge Problem:

The flow of phreatic water in an unconfined aquifer above an impervious base is complicated by two factors: a change in the saturated thickness accompanying the variation in piezometric level, and the presence of recharge by rainfall (Fig. 3).

With the notation of the equations of flow becomes

Darcy $q = -kh\dfrac{dh}{dx}$ Continuity $\dfrac{dq}{dx} = P$

Integrated $q = Px + C_1$

Combined $hdh = -\left(\dfrac{Px + C_1}{k}\right)dx$

Integrated $h^2 = -\dfrac{P}{k}x^2 - \dfrac{2C_1}{k}x + C_2$

For the recharge scheme of Fig. 4 again consisting of three wells fully penetrating the saturated thickness of the aquifer, these boundary conditions give

x = 0 $\quad h^2 = h_n^2 = C_2$

x = 0 $\quad q = -q_0 = C_1$

From which follows x = L

$$h_0^2 = -\frac{P}{k}L^2 + \frac{2q_0}{k}L + h_n^2 \quad \text{------------- (1)}$$

By the quadratic form of this equation the drawdown can be determine as $s_0 = h_0 - h_n$

4. Numerical Modelling of Unconfined Aquifer:

For verifying dimension (shape) of water mound below recharge area, by trial & error varying all the variables like Rainfall (P), permeability (k), distance between two recharge well (L), thickness of saturated soil strata (H), depth of G.W.L. (h_n) in Equation :1 [3] Variable: Rainfall (P) (Fig. 5) Others parameters are fixed (q_0 = 5 x 10^{-5} m^3/m/sec, k= 0.15x10^{-3} m/sec, L=70 m, h_n=10 m, porosity p= 0.3)

Main equation: $h_0^2 = -\frac{P}{k}L^2 + \frac{2q_0}{k}L + h_n^2$

- Detention time $T_{days} = \frac{pHL}{q_0}$
- Recharge rate q_r = (P x L)
- Draw down $s_0 = h_0 - h_n$

Table 1 show that rainfall increases, height of water mound decreases. Drawdown also decreases. It means water percolates through aquifer and merges with ground water. Fig. 5 indicates ABCD zone for Surat city maximum to minimum rainfall 10mm/hr to 5mm/hr respectively. Keeping discharge rate 5x10^{-5} m^3/m/sec constant; water head placed 10.06 m and 7.48 m respectively. Fig. 5 shows that ABCD Zone is the storage of water during recharges and water detent in aquifer about 42 to 48 days. (k = 10^{-3} m/sec, L = 70 m, h_n = 10 m, q_0 = 5x10^{-5} m^3/m/sec, variables are rainfall & recharge rate). Fig. 5 shows that given value of k and L rainfall more than 10 mm/hr, h_0 become negative. So either modified L or k, but for a particular soil aquifer k is fixed then only change in L is possible.

5. Correlations Between Radius of Bore Well (r) and Depth of Soil Strata Above G.W.L (h) with Capacity of Borehole (q_r) (fig. 6):

Overall capacity of Borehole [6] Q_r = k A i t
Sand permeability 10^{-3} m/sec (3.6 m/hr)
Area of Borehole = 2πrh
Hydraulic gradient i = h/L = 15/30
Time = 1 hr
Q_r =3 x (2πrh) x h/L x 1
=3 x (2πrh) x 0.5 x 1
= 9.42 r h
Q_r ≅ 10 x r x h Table 2 shows capacity of Borehole is directly varies with radius of bore well (r) and thickness of pervious strata (h)

6. Design Aspect of Estimate the Recharge Flow (Q_r):

Flow q_r by constant head recharge in borehole. [1]
q_r = 2.75 x d x h x k
d = diameter of bore (m)
h = depth of pervious strata (m)
k = co-efficient of permeability (m/hr)
Q_r = 65 x d x k
Table 3 shows by varying diameter & permeability; estimate the rate of recharge flow.

Case Studies:

1. Evaluation of k Value at Garden Silk Mills Vareli, Surat: [8] (Fig. 7)

Assumptions:
Recharge rate q = 10 m^3/hr, Influence range L = 30 m

$$q = \frac{(h^2 - h_w^2)k}{2L}$$

$$10 = \frac{(30^2 - 28^2)k}{2\times 30}$$

$$k = \frac{10\times 2\times 30}{2\times 58}$$

k = 5 m/hr = 1.4 x 10^{-3} m/sec.
Remark: Site having fine sand as per soil report and it is confirmed with value of permeability after installation of recharge well.

2. Terrance of Raw House at Sarjan Society, Surat (year 2004): Calculation of Recharge Rate from Roofing System:

10 m x 10 m x 0.6 m/year = 60 m^3/year
= 60 x 1000 = 60,000 litters/year
Consider 10 m^3 losses, so 50 m^3 available / year
Water from roofing = 50 m^3 / 100 Sq.m.
≈ 50,000 liter / 100 Sq. m.

3. Panas Recharge Bore Well: S.M.C. 12-7-2000 **(Fig. 8):** 1.5 m depth and 12 m wide tank 100 mm radius of P. V. C. pipe, 12m - 20m sloughed pipes ,20 - 22 cm Gravel pack Recharge rate[8]

$q_r = 5.5 \times r \times h \times k_{av}$

$= 5.5 \times 0.1 \times 18 \times 10^{-3}$

$= 35.6\ m^3/hr$ Say $30\ m^3/hr$.

4. S.V.R. College of Engg. & Tech, Surat (Year 2000):

Ground Water Recharge Project: Technique adopted – Recharge Well & Bore. [7]

Table 4.1 & 4.2 shows quality of Groundwater and increase in level of Groundwater.

5.Radhe Krishna Market, Surat (Year 2005) (Fig. 9):

Evaluation of permeability of soil (k) [4]

$$q_r = \frac{\left(h^2 - h_w^2\right)}{2L} k$$

(Assumed data $q_r = 10\ m^3/hr$, $L = 30$ m)

$$10 = \frac{[12.3^2 - 6.5^2]}{2 \times 30} q_r k$$

$$k = \frac{10 \times 2 \times 30}{18.8 \times 5.8} = 5.5\, m/hr \quad k \approx 1.5 \times 10^{-3}\ m/sec$$

Remark: As per soil data report test data permeability of soil (k) is confirm (coarse sand) with above calculation

Concluding Remark:

- Results of the numerical modeling revealed that a reduction in the growth of the ground-water mound due to decrease in the recharge rate or due to reduction in the radius of recharge basin is most pronounced below and in the vicinity of recharge basin and its magnitude decreases with distance away from the center of the basin. On the other hand, an increase in K value causes significant reduction in the growth of water-table in regions below and adjacent to the basin only. However, an increase in the growth of the water-table of very small magnitude simultaneously occurs in the farther region.
- Table 2 gives design concept of radius r of recharge well with knowing actual value of depth of pervious strata h for the proposed site. If we design a system for higher rate of recharge than install one larger diameter well instead of two smaller dia. well. So it economizes the project cost.
- Table 3 shows recharge rate Q_r of well directly varies with d and k. also shows that with small variation in aquifer permeability, the recharge rate is drastically changed. If permeability of the aquifer is known than only proper diameter of well can be select from given table.

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Table 1: *Calculation of Water Head h_o*

Rainfall (P)		h_0 (m)	S_0 (m)	T (days)	q_r $m^3/m/sec$
m/sec	mm/hr				
$1.1x10^{-9}$	$3.96x10^{-3}$	12.11	2.11	53.72	$7.7x10^{-8}$
$1.2x10^{-8}$	0.0432	12.09	2.09	53.69	$8.4x10^{-7}$
$2.5x10^{-8}$	0.090	12.07	2.07	53.64	$1.05x10^{-6}$
$3.17x10^{-8}$	0.114	12.06	2.06	53.61	$2.2x10^{-6}$
$1.1x10^{-7}$	0.396	11.96	1.96	53.38	$7.7x10^{-6}$
$1.5x10^{-7}$	0.54	11.90	1.90	53.24	$1.1x10^{-5}$
$1.2x10^{-6}$	4.16	10.43	0.43	49.64	$8.4x10^{-5}$
$1.3x10^{-6}$	5.00	10.06	0.06	48.00	$9.1x10^{-5}$
$1.7x10^{-6}$	6.25	9.48	-0.52	47.34	$1.19x10^{-4}$
$2.8x10^{-6}$	10.00	7.48	-2.52	42.00	$1.96x10^{-4}$

$$h_0^2 = - 3.3 \times 10^7 P + 146.67$$

Table 2: $Q_r = 10 \times r \times h$

h (m) / r (m)	4m	5m	6m	8m	10m
Qr m^3/hr					
0.05m	2	2.5	3	4	5
0.075	3	3.75	4.5	6	7.5
0.1	4	5	6	8	10
0.15	6	7.5	9	12	15
0.45	18	22	27	36	45

Table 3: $Q_r = 65 \times d \times k$

k (m/sec) / d (m)	1×10^{-4} m/sec (medium sand)	1×10^{-3} m/sec (Coarse Sand)
Qr m^3/hr		
0.5	12	124
0.6	15	149
0.9	22	223
1.2	30	297

Table 4.1: *Old Recharge Well*

No.	Detail	Before Recharge	One Year after Recharge	Two Year after Recharge
1.	Depth of G.W.L.	10.67m	10m	9.5m
2.	pH	8.2	6.8	7.5
3.	Chloride	550 mg/l	90 mg/l	30 mg/l
4.	Hardness	399 mg/l	200 mg/l	200 mg/l

Table 4.2: *New Recharge Well*

No.	Detail	Before Recharge	One Year after Recharge	Two Year after Recharge
1.	Depth of G.W.L.	8.5m	8.3m	7m
2.	pH	7.5 – 8	7.5	7.0 – 7.5
3.	Chloride	1880 mg/l	30 mg/l	810 mg/l
4.	Hardness	1050 mg/l	100 mg/l	900 mg/l

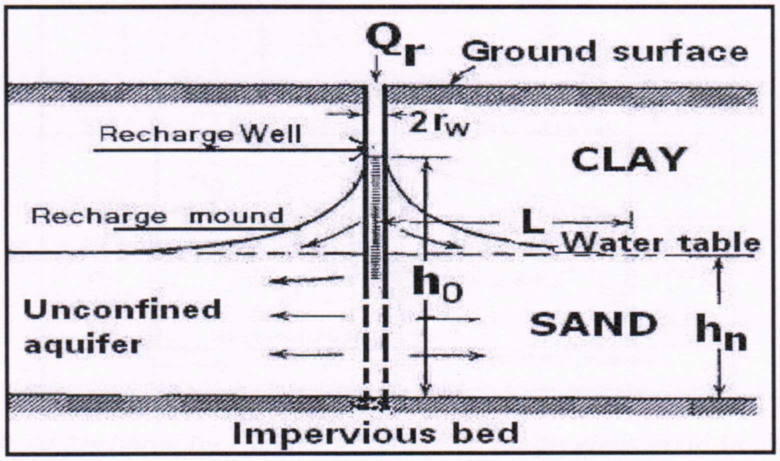

Figure 1: *Aquifer and Recharge Well*

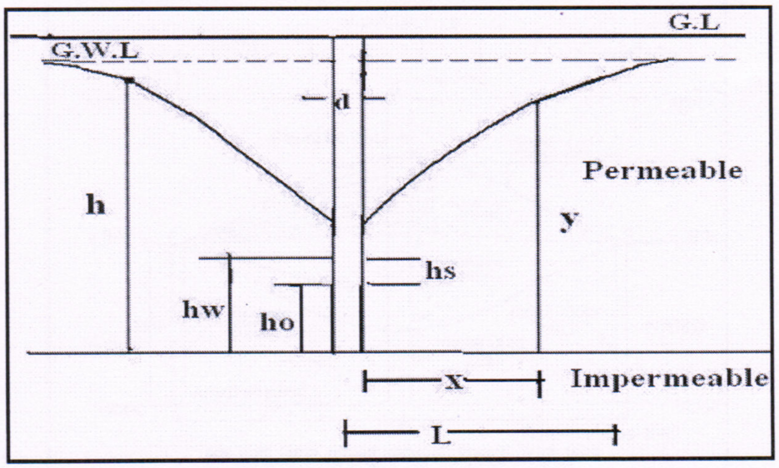

Figure 2: *Fully Penetrating Recharge Well in Unconfined (Gravity flow) Aquifer*

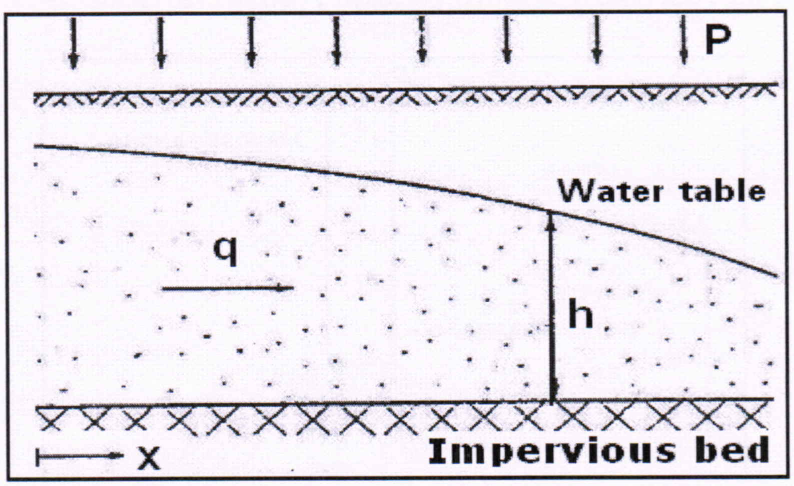

Figure 3: *One Dimensional Flow in an Unconfined Aquifer above an Impervious Base*

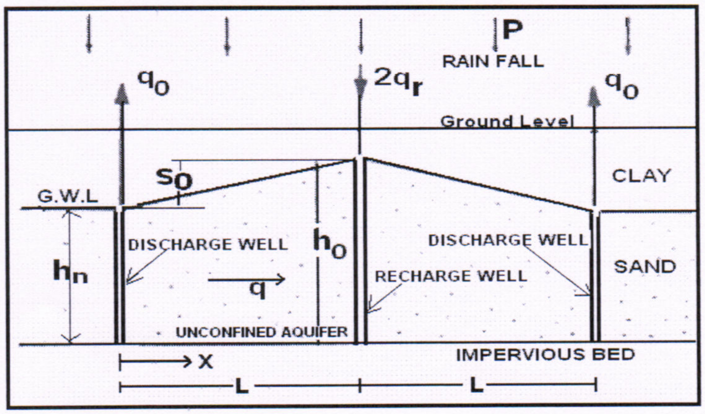

Figure 4: *Artificial Recharge by Fully Penetrating Recharge Wells in an Unconfined Aquifer above an Impervious Base*

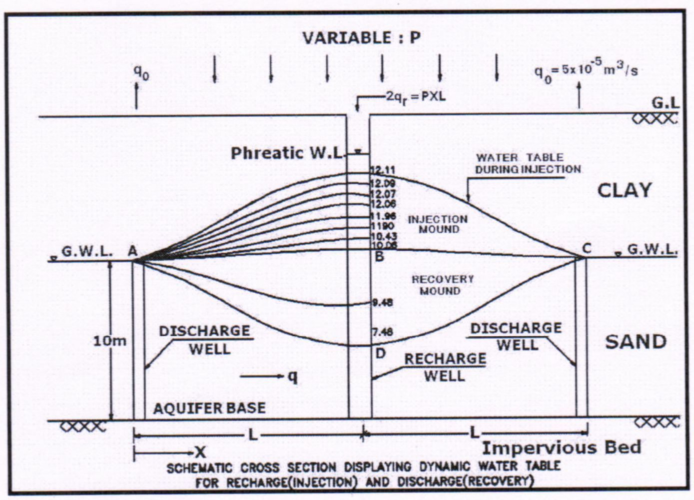

Figure 5: *Schematic Cross Section Displaying Dynamic Water Table for Recharge & Discharge*

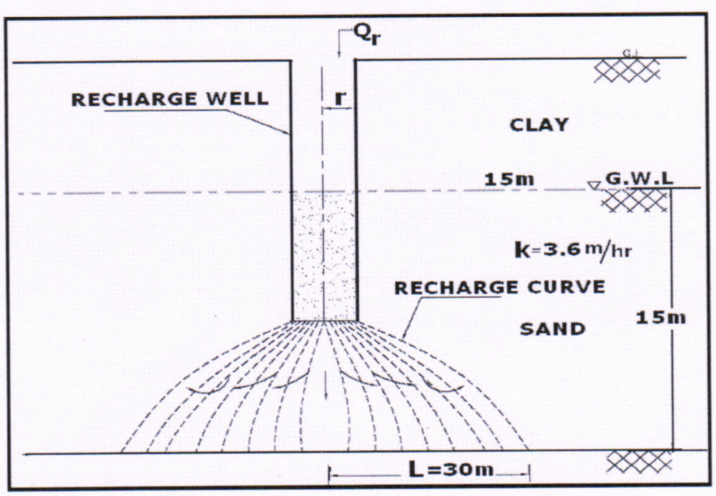

Figure 6: *Installed Recharge Bore Well*

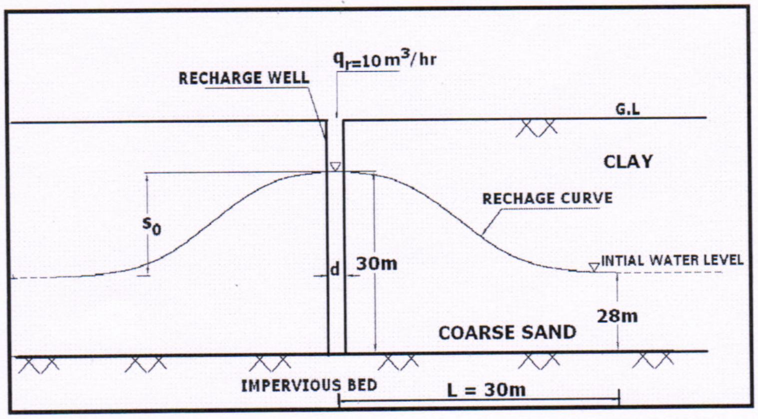

Figure 7: *Recharge Well at GARDEN Silk Milk*

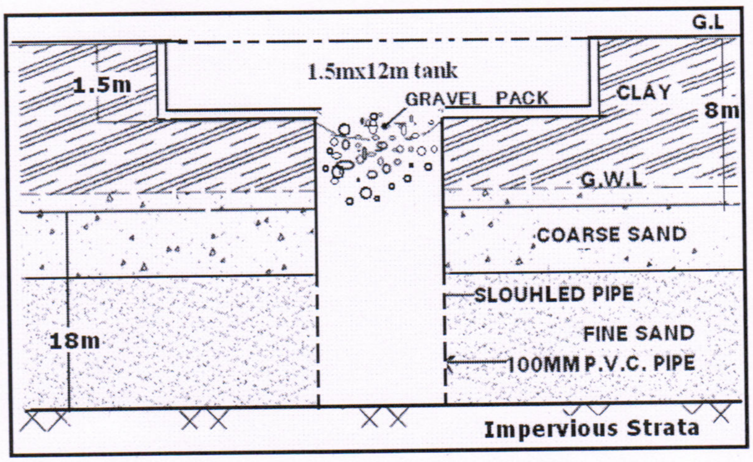

Figure 8: *Recharge Well at Panas*

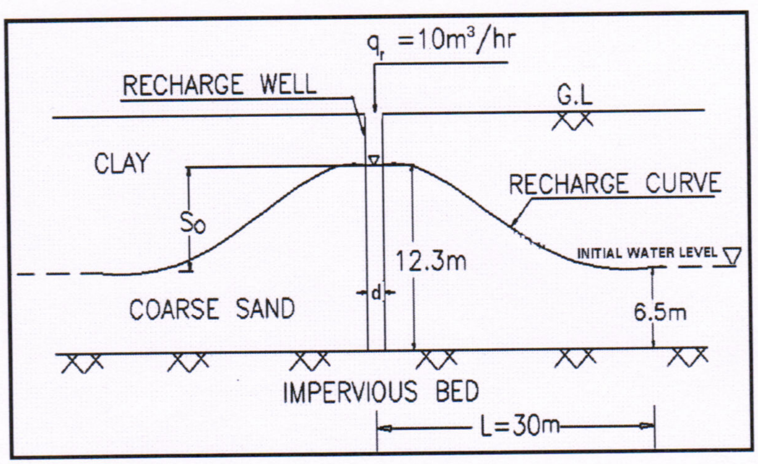

Figure 9: *Recharge Well Installed at Radhe Krishna Market*